ABSTRACT ALGEBRA I

MATH 4110/8116

Course Description:

An introduction to group theory. Various classes of groups are studied: symmetric groups, abelian, cyclic, and permutation groups. Basic tools are developed and used: subgroups, normal subgroups, cosets, the Lagrange theorem, group homomorphisms, quotient groups, direct products, and group actions on a set. The course culminates with the Sylow theorems in finite group theory. The theory is illustrated with examples from geometry, linear algebra, number theory, crystallography, and combinatorics. **3 credits**

Prerequisites:

Undergraduate and Graduate: MATH 4050/8056 with a C- or better or MATH 4560/8566 with a C- or better or permission of instructor

Overview of Content and Purpose of the Course:

 This course covers the basic ideas and applications of group theory. Starting with the symmetries of regular polyhedra in geometry, the course considers various other classes of group (abelian, cyclic, permutation groups and matrix groups). The theory begins with an examination of subgroups, their cosets, and the Lagrange theorem. It is developed more fully with a study of normal subgroups, quotient groups and group homomorphisms. Larger groups are then constructed using direct products. The theory culminated with group actions on a set, and the famous Sylow theorems. The theory and its applications are illustrated with examples from geometry, linear algebra, number theory, crystallography, and combinatorics.

Major Topics:

1) Preliminaries
    a. Modular Arithmetic
    b. Mathematical Induction
    c. Equivalence Relations
    d. Functions (Mappings)

2) Introduction to Groups
    a. The Dihedral Groups

3) Groups
    a. Definition and Examples of Groups
    b. Elementary Properties of Groups

4) Finite Groups; Subgroups
   a. Terminology and Notation
   b. Subgroup Tests
   c. Examples of Subgroups

5) Cyclic Groups
   a. Properties of cyclic Groups
   b. Classification of Subgroups of Cyclic Groups

6) Permutation Groups
   a. Definition and Notation
   b. Properties of Permutations

7) Isomorphisms
   a. Definition and Examples
   b. Cayley's Theorem
   c. Properties of Isomorphisms
   d. Automorphisms

8) Cosets and Lagrange's Theorem
   a. Properties of Cosets
   b. Lagrange's Theorem and Consequences

9) External Direct Products
   a. Definition and Examples
   b. Properties of External Direct Products
   c. The Group of Units Modulo n as an External Direct Product

10) Normal Subgroups and Factor Groups
   a. Normal Subgroups
   b. Factor Groups
   c. Applications of Factor Groups

11) Group Homomorphisms
   a. Definition and Examples
   b. Properties of Homomorphisms
   c. The First Isomorphism Theorem

12) Fundamental Theorem of Finite Abelian Groups
   a. The Fundamental Theorem
   b. The Isomorphism Classes of Abelian Groups

13) Sylow Theorems
   a. Conjugacy Classes
   b. The Class Equation
   c. The Sylow Theorems
   d. Applications of Sylow Theorems

14) Finite Simple Groups
   a. Historical Background
   b. Nonsimplicity Tests
   c. The Simplicity of A_5

15) Generators and Relations
   a. Definitions and Notation
   b. Free Group
   c. Generators and Relations
   d. Classification of Groups of Order Up to 15
   e. Characterization of Dihedral Groups

16) Symmetry Groups
   a. Isometries
   b. Classification of Finite Plane Symmetry Groups
   c. Classification of Finite Groups of Rotations in $\mathbf{R}^3$

Textbook:

 Gallian, Joseph A. *Contemporary Abstract Algebra, 7th ed*. Belmont: Brooks/Cole, 2009.

                                                              May 2017